

MENIIT

NEET | IIT-JEE | FOUNDATION

Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | **Web:** www.meniit.com

JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 26-08-2021 (MORNING SHIFT) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

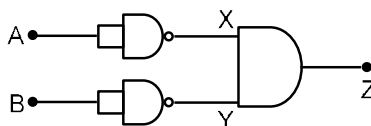
**QUESTION
&
SOLUTIONS**

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

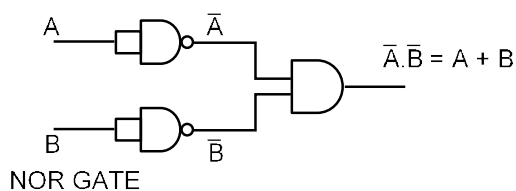
1. Identify the logic operation carried out by the given circuit



- (1) NOR (2) OR (3) NAND (4) AND

Ans. (1)

Sol.

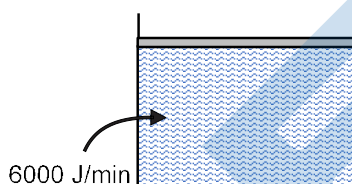


2. An electric appliance supplies 6000J/min heat to the system. If the system delivers a power of 90 W. How long it would take to increase the internal energy by 2.5×10^3 J ?

- (1) 4.1×10^1 s (2) 2.4×10^3 s (3) 2.5×10^2 s (4) 2.5×10^1 s

Ans. (3)

Sol.



$$= \frac{6000}{60} \text{ J/s} = 100 \text{ J/s}$$

$$\Delta U = Q - w$$

$$2500 = 100t - 90t$$

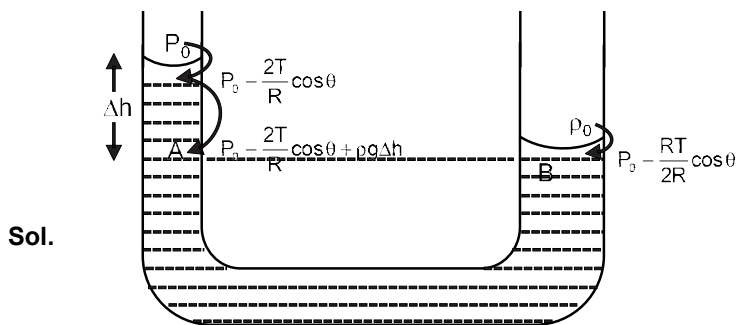
$$t = 250 \text{ s}$$

3. Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in level of two limbs of the tube?

[Take surface tension of water $T = 7.3 \times 10^{-2} \text{ Nm}^{-1}$, angle of contact = 0, $g = 10 \text{ ms}^{-2}$ and density of water = $1.0 \times 10^3 \text{ kg m}^{-3}$]

- (1) 4.97 mm (2) 2.19 mm (3) 5.34 mm (4) 3.62 mm

Ans. (2)



Sol.

$$P_0 - \frac{2T}{R_1} \cos \theta - h_1 \rho g = P_0 - \frac{2T}{R_2} \cos \theta - h_2 \rho g$$

$$(h_1 - h_2) \rho g = 2T \left(\frac{1}{R_2} - \frac{1}{R_1} \right) \cos \theta$$

$$h_1 - h_2 = \frac{2 \times 7.3 \times 10^{-2} \times \cos \theta}{1000 \times \left(\frac{1}{10} - \frac{1}{8} \right)} = \frac{3}{20} = 2.19 \text{ mm}$$

4. In a Screw Gauge, fifth division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation the reading on the main scale is 5 mm and the 20th division of the circular scale coincides with reference line. Calculate the true reading.

- (1) 5.00 mm (2) 5.20 mm (3) 5.25 mm (4) 5.15 mm

Ans. (4)

Sol. L.C. of screw Gauge = $\frac{0.5}{50} = 0.01 \text{ mm}$

Zero error of screw Gauge = $5 \times \text{L.C.} = 5 \times 0.01 = 0.05 \text{ mm}$

Reading of screw gauge = main scale reading + circular scale reading \times L.C. – Zero error
 = $5 + 20 \times 0.01 - 0.05 = 5.15 \text{ mm}$

5. Car B overtakes another car A at relative speed of 40 ms^{-1} . How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from car A?

- (1) 0.2 ms^{-1} (2) 40 ms^{-1} (3) 0.1 ms^{-1} (4) 4 ms^{-1}

Ans. (3)

Sol. $u = -1.9 \text{ m}$

$f = 10 \text{ cm}$

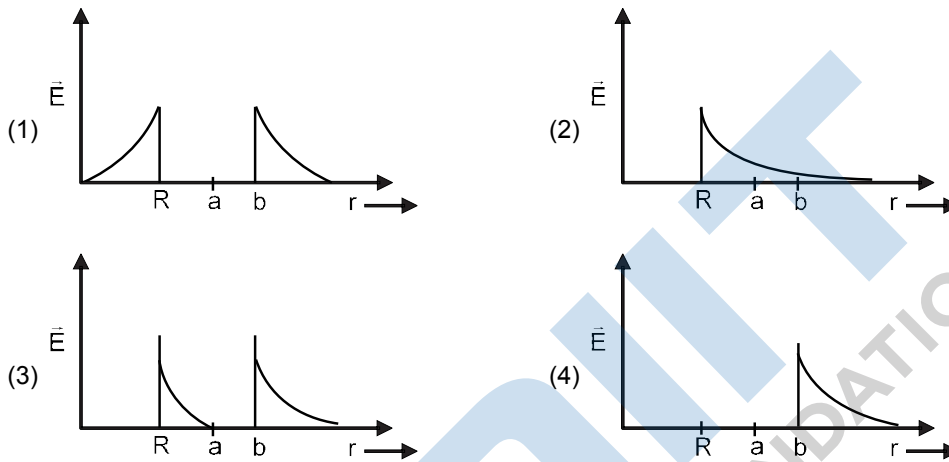
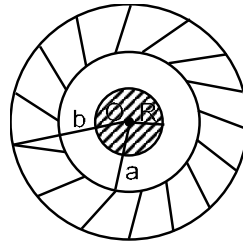
$$m = \frac{f}{f - u} = \frac{10}{10 - (-190)}$$

$$m = \frac{10}{200} = \frac{1}{20}$$

$$V_i = m^2 V_o = \left(\frac{1}{20} \right)^2 \times 40$$

$V_i = -0.1 \text{ m/s}$

6. A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and outer radius b as shown in figure. The approximate variation electric field \vec{E} as a function of distance r from centre O is given by:



Ans. (3)

Sol. $0 < r < R$

$$E = 0$$

$R \leq r \leq a$

$$E = \frac{kQ}{r^2}$$

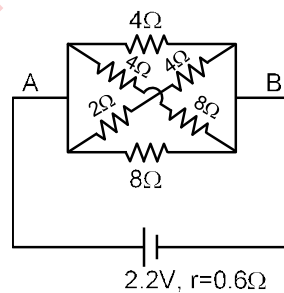
$a < r \leq b$

$$E = 0$$

$r > b$

$$E = \frac{kQ}{r^2}$$

7. In the given figure, the emf of the cell is 2.2 V and if internal resistance is 0.6Ω . Calculate the power dissipated in the whole circuit :



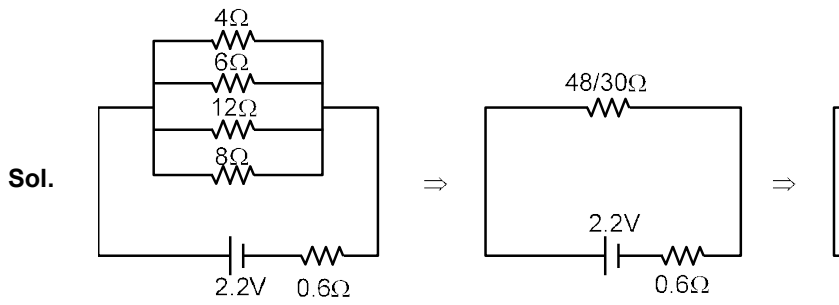
(1) 0.65 W

(2) 4.4 W

(3) 1.32 W

(4) 2.2 W

Ans. (4)



$$P = \frac{V^2}{R}, P = \frac{(2.2)^2}{2.2} = 2.2W$$

8. An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8 A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds :

- (1) 0.8 (2) 0.125 (3) 0.4 (4) 0.2

Ans. (4)

Sol. $U = \frac{1}{2}LI^2$

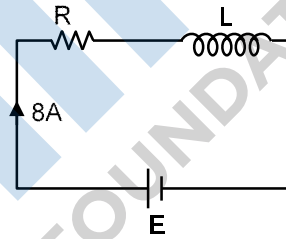
$$64 = \frac{1}{2} \cdot L \cdot (8)^2 \quad L = 2H$$

and $P = I^2R$

$$640 = (8)^2 \cdot R \Rightarrow R = 10\Omega$$

time constant

$$\frac{L}{R} = \frac{2}{10} = 0.2$$



9. The material filled between the plates of a parallel plate capacitor has resistivity $200 \Omega m$. The value of capacitance of the capacitor is $2pF$. If a potential difference of 40 V is applied across the plates of the capacitor, then the value of leakage current following out of the capacitor is: (given the value of relative permittivity of material is 50)

- (1) $9.0 \mu A$ (2) $9.0 mA$ (3) $0.9 mA$ (4) $0.9 \mu A$

Ans. (3)

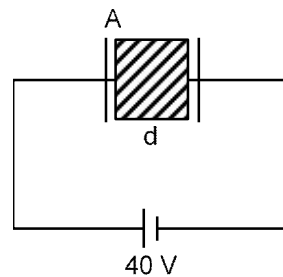
Sol. $\rho = 200 \Omega m$

$$K = 50$$

$$C = \frac{\epsilon_0 K A}{d} = 2 \cdot 10^{12} f$$

$$R = \frac{d}{A \rho}$$

$$I = \frac{V}{R} = \frac{AV}{d} = \frac{V}{\rho} \cdot \frac{C}{K} = \frac{40}{200} \cdot \frac{2 \cdot 10^{12}}{8.85 \cdot 10^{12} \cdot 50} = 0.9mA$$



10. In a photoelectric experiment ultraviolet light of wavelength 280 nm used with lithium cathode having work function $\phi = 2.5$ eV. If the wavelength of incident light is switched to 400 nm, find out the change in the stopping potential. ($h = 6.63 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms⁻¹)
- (1) 0.6 V (2) 1.1 V (3) 1.3 V (4) 1.9 V

Ans. (3)

Sol. $eV_0 = \frac{hc}{\lambda}$

$$eV_1 = \frac{hc}{280\text{nm}}$$

$$eV_2 = \frac{hc}{400\text{nm}}$$

$$e(V_1 - V_2) = 1240 \left(\frac{1}{280} - \frac{1}{400} \right) = 1.3\text{eV}$$

11. The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that rocket is given an acceleration of 20 ms⁻². The gases come out at a relative speed of 500 ms⁻¹ with respect to the rocket: [Use $g = 10$ m/s²]
- (1) 6.0×10^2 kg s⁻¹ (2) 500 kg s⁻¹ (3) 10 kg s⁻¹ (4) 60 kg s⁻¹

Ans. (4)

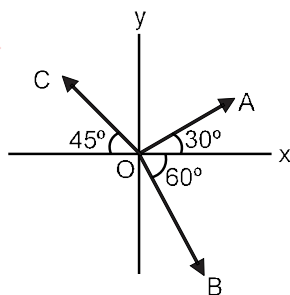
Sol. $f_r mg = ma$

$$\frac{dm}{dt} v_r = mg + ma$$

$$\frac{dm}{dt} 500 = 1000(10 + 20)$$

$$\frac{dm}{dt} = 60\text{Kg/s}$$

12. The magnitude of vector \vec{OA} , \vec{OB} and \vec{OC} in the given figure are equal. The direction of \vec{OA} , \vec{OB} , \vec{OC} with x-axis will be:



- (1) $\tan^{-1} \frac{1}{1} \frac{\sqrt{3}}{\sqrt{3}} \frac{\sqrt{2}}{\sqrt{2}}$ (2) $\tan^{-1} \frac{1}{1} \frac{\sqrt{3}}{\sqrt{3}} \frac{\sqrt{2}}{\sqrt{2}}$ (3) $\tan^{-1} \frac{\sqrt{3}}{1} \frac{1}{\sqrt{3}} \frac{\sqrt{2}}{\sqrt{2}}$ (4) $\tan^{-1} \frac{\sqrt{3}}{1} \frac{1}{\sqrt{3}} \frac{\sqrt{2}}{\sqrt{2}}$

Ans. (2)

Sol. $\vec{OA} = R \cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}$

$\vec{OB} = R \cos 60^\circ \hat{i} + (\sin 60^\circ) \hat{j}$

$\vec{OC} = R \cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}$

$\vec{OA} + \vec{OB} + \vec{OC} = R \left[\frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \right] \hat{i} + R \left[\frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \right] \hat{j}$

angle with the x-axis

$$\tan \theta = \frac{\frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}{\frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}$$

$$\tan^{-1} \frac{1 + \sqrt{3} + \sqrt{2}}{1 + \sqrt{3} + \sqrt{2}}$$

13. The fractional change in the magnetic field intensity at distance 'r' from centre on the axis of current carrying coil of radius 'a' to the magnetic field intensity at centre of the same coil is : (Take $r < a$)

(1) $\frac{2r^2}{3a^2}$

(2) $\frac{3r^2}{2a^2}$

(3) $\frac{2a^2}{3r^2}$

(4) $\frac{3a^2}{2r^2}$

Ans. (2)

Sol. $B_{\text{axis}} = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$

$B_{\text{centre}} = \frac{\mu_0 i}{2R}$

$B_{\text{axis}} = \frac{\mu_0 i a^2}{2(a^2 + r^2)^{3/2}}$

\therefore fractional change in magnetic field $= \frac{\frac{\mu_0 i}{2a} \frac{\mu_0 i a^2}{2(a^2 + r^2)^{3/2}}}{\frac{\mu_0 i}{2a}} - 1 = \frac{1}{1 + \frac{r^2}{a^2}^{3/2}} - 1$

$= 1 - \frac{3r^2}{2a^2} = \frac{3r^2}{2a^2}$

14. What equal length of an iron wire a copper-nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3 Ω?

[Given resistivities of iron and copper-nickel alloy wire are 12 μΩ cm and 51 μΩ cm respectively]

(1) 82 m

(2) 97 m

(3) 110 m

(4) 90 m

Ans. (2)

Sol. $\frac{R_1 R_2}{R_1 + R_2} = 3$

$$\frac{1}{2} \frac{\ell}{A} 3 ; \ell 3 A \frac{(1/2)}{1/2} 3 \frac{(10^3)^2}{12 \cdot 51 \cdot 10^{16}} 63 \cdot 10^8 ; \ell 97m$$

15. A series LCR circuit driven by 300 V at frequency of 50 Hz contains a resistance $R = 3 \text{ k}\Omega$, an inductor of inductive reactance $X_L = 250 \pi \Omega$ and an unknown capacitor. The value of capacitance to maximize the average power should be: (take $\pi^2 = 10$)

- (1) $4 \mu\text{F}$ (2) $400 \mu\text{F}$ (3) $40 \mu\text{F}$ (4) $25 \mu\text{F}$

Ans. (1)

Sol. At resonance $\omega = 2\pi f$
 $X_C = X_L = 2\pi \times 50$
 $\frac{1}{C} L = 100 \pi$
 $\frac{1}{C} 250 = 100 \pi$
 $C = \frac{1}{100 \cdot 250} 4\mu\text{F}$

16. The rms speed of the molecules of Hydrogen, Oxygen and Carbon dioxide at the same temperature are V_H, V_O and V_C respectively then :

- (1) $V_H > V_O > V_C$ (2) $V_H = V_O > V_C$ (3) $V_C > V_O > V_H$ (4) $V_H = V_O = V_C$

Ans. (1)

Sol. $V_{\text{rms}} = \sqrt{\frac{3RT}{M}} \frac{1}{\sqrt{M}}$
 $M_{H_2} \quad M_{O_2} \quad M_{CO_2}$
 $V_{H_2} \quad V_{O_2} \quad V_{CO_2}$

17. A particular hydrogen like ion radiation of frequency $2.92 \times 10^{15} \text{ Hz}$ when it makes transition from $n = 3$ to $n = 1$. The frequency in Hz of radiation emitted in transition from $n = 2$ to $n = 1$ will be:

- (1) 6.57×10^{15} (2) 2.46×10^{15} (3) 4.38×10^{15} (4) 0.44×10^{15}

Ans. (2)

Sol. $h \nu = E_0 z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$
 $\frac{1}{n_1^2} - \frac{1}{n_2^2}$
 $\frac{2.19 \cdot 10^{15}}{h} = \frac{1}{1} - \frac{1}{9}$
 $\frac{1}{1} - \frac{1}{4}$

$$2.19 \times 10^{15} \frac{3}{4} \frac{9}{8}$$

$$\nu = 2.46 \times 10^{15} \text{ Hz}$$

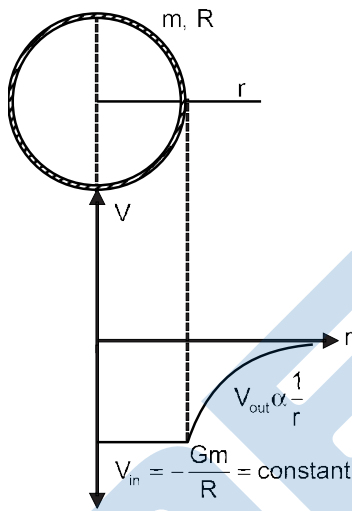
18. Inside a uniform spherical shell:
- (a) The gravitation field is zero
 - (b) The gravitational potential is zero
 - (c) The gravitational field is same everywhere.
 - (d) the gravitational potential is same everywhere.
 - (e) all of the above

Choose the most appropriate answer from the options given below:

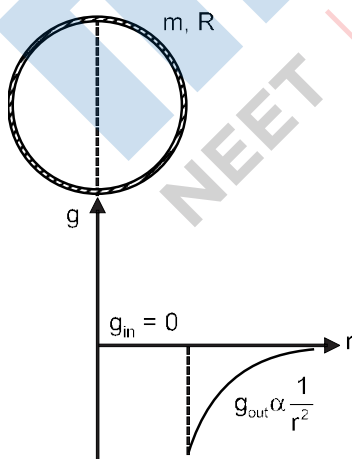
- (1) (b), (c), and (d) only
- (2) (e) only
- (3) (a), (c) and (d) only
- (4) (a), (b) and (c) only

Ans. (3)

Sol. Gravitational potential due to uniform spherical shell



Gravitational field due to uniform spherical shell



19. If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimension of P in the formula $P = EL^2M^{-5}G^{-2}$ are:
- (1) $[M^1L^1T^{-2}]$ (2) $[M^{-1}L^{-1}T^2]$ (3) $[M^0L^1T^0]$ (4) $[M^0L^0T^0]$

Ans. (4)

Sol. $P = EL^2M^{-5}G^{-2}$

$$P = [ML^2T^{-2}] [ML^2T^{-1}]^2 [M^{-5}] [M^{-1}L^3T^{-2}]^{-2}$$

$$P = [M^0L^0T^0]$$

20. **Statement-I** : By doping semiconductor with pentavalent material, the density increases.

Statement-II : The n-type semiconductor has net negative charge.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Both Statement I and Statement II are false. (2) Both Statement I and Statement II are true.
(3) Statement I is false but Statement II is true. (4) Statement I is true but Statement II is false.

Ans. (4)

Sol. Pentavalent material have excess free e^- So e^- density increases but overall semiconductor is neutral.

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. A source and a detector move away from each other in absence of wind with a speed of 20 m/s with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be ____ Hz.

Ans. (2025)

Sol. $f' = f \frac{v - v_0}{v - v_s}$

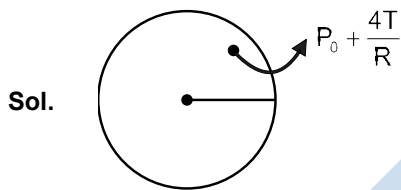
$$1800 = f \frac{340 - 20}{340 - 20}$$

$$f = \frac{1800 \times 360}{320}$$

$$f = 2025 \text{ Hz}$$

2. A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is ____ cm.

Ans. (2)



$$P_0 + \frac{4T}{6} = P_0 + \frac{4T}{3} = P_0 + \frac{4T}{R}$$

$$\frac{1}{6} = \frac{1}{3} - \frac{1}{R}$$

$$R = 2 \text{ cm}$$

3. An amplitude modulated wave is represented by $C_m(t) = 10 (1 + 0.2 \cos 12560t) \sin (111 \times 10^4 t)$ volts. The modulating frequency in kHz will be ____.

Ans. 2

Sol. $C_m = A_c [1 + \frac{A_m}{A_c} \sin 2\pi f_m t]$

$$2\pi f_m = 12560$$

$$f_m = 2000 \text{ Hz} = 2 \text{ KHz}$$

4. The electric field in a plane electromagnetic wave is given by

$\vec{E} = 200 \cos \left(\frac{0.5 \times 10^3}{m} x - 1.5 \times 10^{11} \frac{\text{rad}}{s} t \right) \frac{V}{m} \hat{j}$. If this wave falls normally on a perfectly reflecting surface having an area of 100 cm^2 . If the radiation pressure exerted by the E. M. wave on the surface during a 10 minute exposure is $\frac{x}{10^9} \frac{N}{m^2}$. Find the value of x.

Ans. 354

Sol. $E_0 = 200$

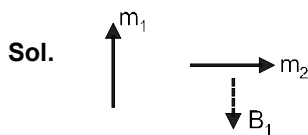
$$I = \frac{1}{2} \epsilon_0 E_0^2 c$$

Radiation pressure

$$P = \frac{2I}{c} = \frac{2}{c} \left(\frac{1}{2} \epsilon_0 E_0^2 c \right) = \epsilon_0 E_0^2 = 8.85 \times 10^{-12} \times 200^2 = 8.85 \times 10^{-8} \times 4 = \frac{354}{10^9}$$

5. Two short magnetic dipoles m_1 and m_2 each having magnetic moment of 1 Am^2 are placed at point O and P respectively. The distance between OP is 1 meter. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is $\text{_____} \times 10^{-7} \text{ Nm}$.

Ans. 1



$$\vec{M}_2 \cdot \vec{B}_1$$

$$M_2 B_1 \sin 90^\circ = 1 \times \frac{\mu_0}{4\pi} \frac{M}{(1)^3} = 1 \times 10^{-7} \text{ Nm}$$

6. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of _____ m. (take $g = 10 \text{ m/s}^2$)

Ans. 50

Sol. $S_1 = S_2$

$$35t - \frac{1}{2} g t^2 = 35(t - 3) - \frac{1}{2} g (t - 3)^2$$

$$35t - \frac{1}{2} g t^2 = 35t - 35 \times 3 - \frac{1}{2} g (t^2 - 6t + 9)$$

$$35 \times 3 + 45 = 30 t$$

$$t = \frac{150}{30} = 5$$

$$\text{height } h = 35 \times 5 - \frac{1}{2} \times 10 \times 5^2$$

$$h = 175 - 125 = 50 \text{ m}$$

7. White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.3 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. The difference in wavelength of red and violet light is ____ nm.

Ans. 300

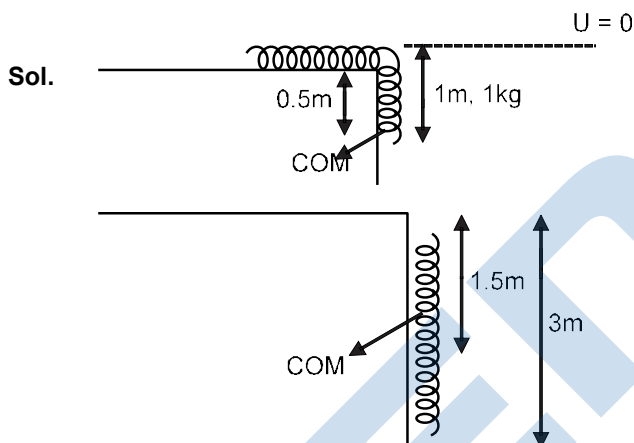
Sol. $y_1 = \frac{\lambda_1 D}{d}$

$y_2 = \frac{\lambda_2 D}{d}$

$\lambda_2 - \lambda_1 = (y_2 - y_1) \frac{d}{D} = (3.5 - 2.0) \text{mm} \frac{0.3 \text{mm}}{1.5 \text{m}} = 300 \text{nm}$

8. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is ____.

Ans. 40



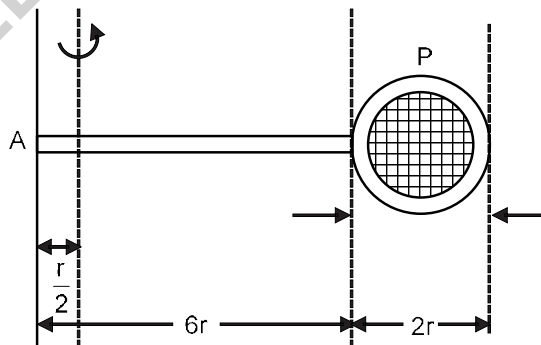
Apply conservation of energy $U_i + K_i = U_f + K_f$

$10 + 0 = 0 + K_f$

K.E. = 45 - 5

K.E. = 40J

9. Consider a badminton racket with length scales as shown in the figure.



If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will be _____ Mr^2 .

Ans. 52

Sol. $I_A = \frac{Mr^2}{2} + M \left(7r + \frac{r}{2}\right)^2 = \frac{M(6r)^2}{12} + M \left(3r + \frac{1}{2}\right)^2$

$$\frac{Mr^2}{2} + M \frac{(169)r^2}{4} = M3r^2 + \frac{M25r^2}{4} = \frac{171Mr^2}{4} = \frac{37Mr^2}{4} = \frac{208}{4}Mr^2 = 52Mr^2$$

10. Two travelling waves produces a standing wave represented by equation. $y = 1.0 \text{ mm} \cos(1.57 \text{ cm}^{-1})x \sin(78.5 \text{ s}^{-1})t$. The node closest to the origin in the region $x > 0$ will be at $x =$ _____ cm.

Ans. 1

Sol. At Node

$$y = 0$$

$$1 \cos(1.57x) \sin(178.5)t = 0$$

$$\cos(1.57x) = 0$$

$$1.57x = \frac{\pi}{2}$$

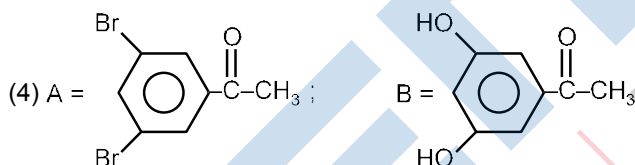
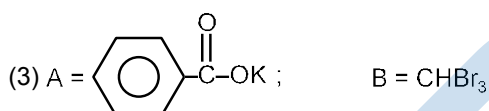
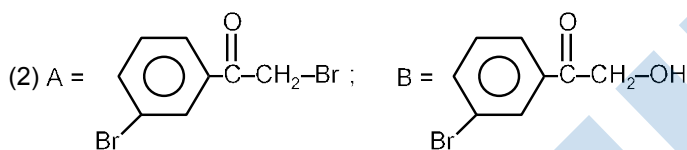
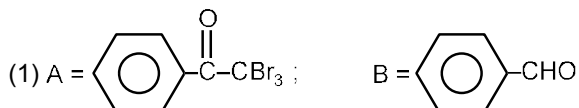
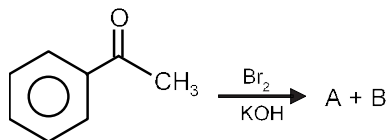
$$x = 1 \text{ cm}$$

PART B : CHEMISTRY

Single Choice Type

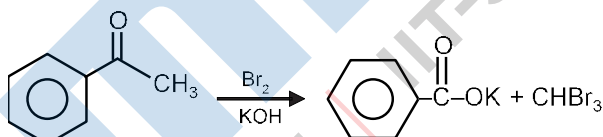
This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The major products formed in the following reaction sequence A and B are :



Ans. (3)

Sol. It is Haloform reaction.



2. Given below are two statements.

Statement-I : The choice of reducing agents for metals extraction can be made by using Ellingham diagram, a plot of ΔG vs temperature.

Statement-II : The value of ΔS increases from left to right in Ellingham diagram.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Statement-I is false but Statement-II is true.
- (2) Both Statement-I and Statement-II are false.
- (3) Statement-I is true but Statement-II is false.

(4) Both Statement-I and Statement-II are true.

Ans. (3)

Sol. Ellingham diagram is used to check which metal oxide is reduced by which compound.

3. Given below are two statements :

Statement-I : In the titration between strong acid and weak base methyl orange is suitable as an indicator.

Statement-II : For titration of acetic acid with NaOH phenolphthalein is not a suitable indicator.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement-I is false but Statement-II is true

(2) Both Statement-I and Statement-II are false

(3) Statement-I is true but Statement-II is false

(4) Both Statement-I and Statement-II are true

Ans. (3)

Sol. (1) MeOH is suitable indicator for titration of strong acid and weak base.

(2) Hph is suitable indicator for titration of weak acid and strong base.

4. What the products formed in sequence when excess of CO_2 is passed in slaked lime ?

(1) CaCO_3 , $\text{Ca}(\text{HCO}_3)_2$

(2) $\text{Ca}(\text{HCO}_3)_2$, CaCO_3

(3) CaO , $\text{Ca}(\text{HCO}_3)_2$

(4) CaO , CaCO_3

Ans. (1)

Sol. Slaked lime $\xrightarrow{\text{Pass } \text{CO}_2}$ CaCO_3 $\xrightarrow{\text{Prolong Passage of } \text{CO}_2}$ $\text{Ca}(\text{HCO}_3)_2$

5. Which one of the following methods is most suitable for preparing deionized water ?

(1) Permutit method

(2) Synthetic resin method

(3) Calgon's method

(4) Clark's method

Ans. (2)

Sol. Pure de-mineralised (de-ionised) water free from all soluble mineral salt is obtained by passing water successively through a cation exchange and an anion exchange resins.



6. Which one of the following when dissolved in water gives coloured solution in nitrogen atmosphere ?

(1) Cu_2Cl_2

(2) CuCl_2

(3) ZnCl_2

(4) AgCl

Ans. (2)

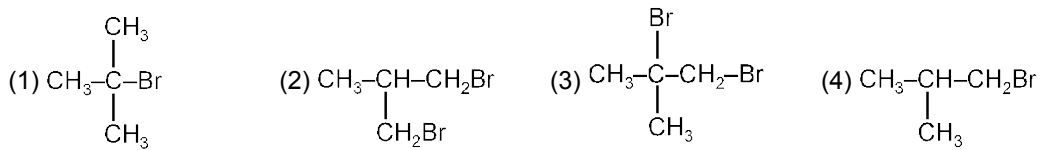
Sol. CuCl_2 dissolve in water & give coloured solutions.

AgCl not soluble in water.

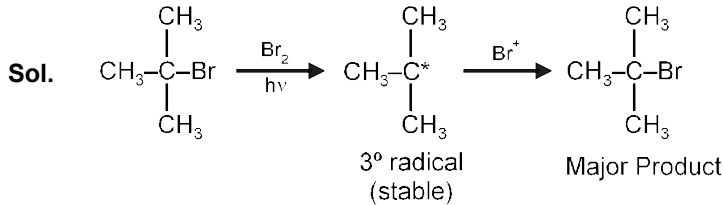
Cu_2Cl_2 is not soluble in water.

ZnCl_2 is soluble but colourless.

7. Excess of isobutene on reaction with Br₂ in presence of light at 125°C gives which one of the following, as the major product ?



Ans. (1)



8. Which one of the following, is correct for the adsorption of a gas at a given temperature on a solid surface?



Ans. (1)

Sol. Adsorption of gas on solid surface is exothermic process & entropy is decrease in this process so $\Delta H < 0, \Delta S < 0$

9. Given below are two statements :

Statement-I : Frenkel defects are vacancy as well as interstitial defects.

Statement-II : Frenkel defect leads to colour in ionic solids due to presence of F-centres.

Choose the most appropriate answer for the statements from the options given below :

- (1) Both Statement-I and Statement-II are false
 (2) Statement-I is true but Statement-II is false
 (3) Statement-I is false but Statement-II is true
 (4) Both Statement-I and Statement-II are true

Ans. (2)

Sol. Frenkel defect is dislocation defect, it is not interstitial defect and it is not responsible for colour of solid.

Note : NTA ans. is (2), But Zigyan ans. is (1).

10. Given below are two statements :

Statement-I : The limiting molar conductivity of KCl (strong electrolyte) is higher compared to that of CH₃COOH (weak electrolyte).

Statement-II : Molar conductivity decreases with decrease in concentration of electrolyte.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Statement-I is true but Statement-II is false
 (2) Both Statement-I and Statement-II are true
 (3) Statement-I is false but Statement-II is true

(4) Both Statement-I and Statement-II are false

Ans. (4)

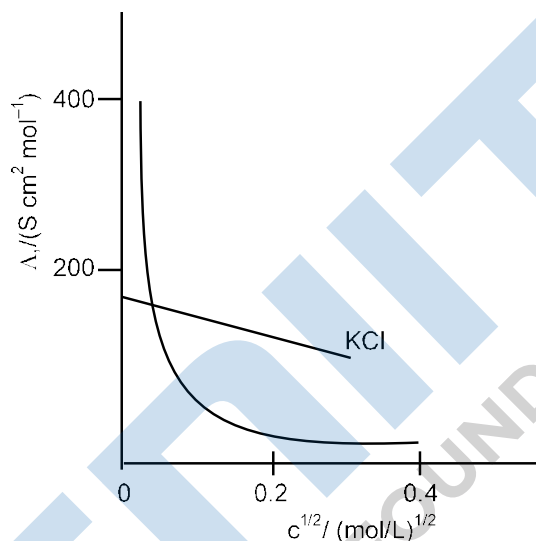
Sol.

Ion	K ⁺	Cl ⁻	CH ₃ COO ⁻	H ⁺
$\lambda^\circ / (\text{S cm}^2 \text{mol}^{-1})$	73.5	76.3	40.9	349.6

$$\lambda^\circ_{\text{M}}(\text{CH}_3\text{COOH}) = \lambda^\circ_{\text{M}}(\text{H}^+) + \lambda^\circ_{\text{M}}(\text{CH}_3\text{COO}^-) = 349.6 + 40.9 = 390.5 \text{ S cm}^2 \text{mol}^{-1}$$

$$\lambda^\circ_{\text{M}}(\text{KCl}) = \lambda^\circ_{\text{M}}(\text{K}^+) + \lambda^\circ_{\text{M}}(\text{Cl}^-) = 73.5 + 76.3 = 149.8 \text{ S cm}^2 \text{mol}^{-1}$$

So, statement-I is false.



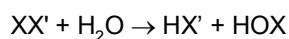
For an electrolyte the dilution increases (or) decrease in concentration, the number of ions increases due to increase of degree of dissociation. So, the molar conductivity increases. Hence Statement-II is false.

11. The incorrect statement is :

- (1) On hydrolysis ClF forms HOCl and HF.
- (2) Cl₂ is more reactive than ClF.
- (3) F₂ is more reactive than ClF.
- (4) F₂ is a stronger oxidizing agent than Cl₂ in aqueous solution.

Ans. (2)

Sol. In general, interhalogen compounds are more reactive than halogens (except fluorine). This is because X - X' bond in interhalogens is weaker than X - X bond in halogens except F - F bond. All these undergo hydrolysis giving halide ion derived from the smaller halogen and a hypohalite (when XX'), halite (when XX₃), halate (when XX₅) and perchalate (when XX₇) anion derived from the larger halogen.



12. The conversion of hydroxyapatite occurs due to presence of F⁻ ions in water. The correct formula of hydroxyapatite is :

- (1) $[3\text{Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_2]$ (2) $[3\text{Ca}(\text{OH})_2 \cdot \text{CaF}_2]$
 (3) $[\text{Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_2]$ (4) $[3\text{Ca}_3(\text{PO}_4)_2 \cdot \text{Ca}(\text{OH})_2]$

Ans. (4)

Sol. Hydroxyapatite $\rightarrow [3\text{Ca}_3(\text{PO}_4)_2 \cdot \text{Ca}(\text{OH})_2]$

Hydroxyapatite in presence of F⁻ ions converts to fluorapatite $[3\text{Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_2]$.

13. Given below are two statements :

Statement-I : According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in positive charges on the nucleus as there is no strong hold on the electron by the nucleus.

Statement-II : According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in principal quantum number.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Statement-I is true but Statement-II is false
 (2) Statement-I is false but Statement-II is true
 (3) Both Statement-I and Statement-II are false
 (4) Both Statement-I and Statement-II are true

Ans. (2)

Sol. $V_n = (2.18 \times 10^6) \frac{Z}{n} \text{ m/sec}; V_n \propto \frac{1}{n}; n \rightarrow V_n$

14. Which one of the following complexes is violet in colour ?

- (1) $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3 \cdot \text{H}_2\text{O}$ (2) $[\text{Fe}(\text{CN})_5\text{NOS}]^{4-}$
 (3) $[\text{Fe}(\text{CN})_6]^{4-}$ (4) $[\text{Fe}(\text{SCN})_6]^{4-}$

Ans. (2)

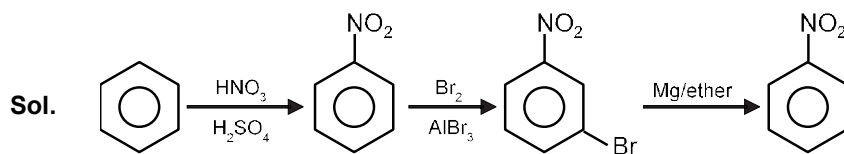
Sol. $\text{S}^{2-} + \text{Na}_2[\text{Fe}(\text{CN})_5(\text{NO})] \rightarrow \text{Na}_4[\text{Fe}(\text{CN})_5(\text{NOS})]$

Sodium nitroprusside Violet colour

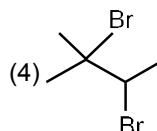
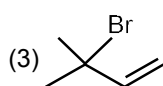
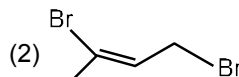
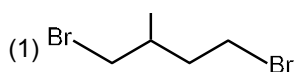
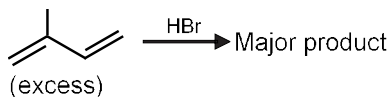
15. The correct sequential addition of reagents in the preparation of 3-nitrobenzoic acid from benzene is :

- (1) $\text{Br}_2/\text{AlBr}_3, \text{HNO}_3/\text{H}_2\text{SO}_4, \text{Mg/ether}, \text{CO}_2, \text{H}_3\text{O}^+$
 (2) $\text{Br}_2/\text{AlBr}_3, \text{NaCN}, \text{H}_3\text{O}^+, \text{HNO}_3/\text{H}_2\text{SO}_4$
 (3) $\text{HNO}_3/\text{H}_2\text{SO}_4, \text{Br}_2/\text{AlBr}_3, \text{Mg/ether}, \text{CO}_2, \text{H}_3\text{O}^+$
 (4) $\text{Br}_2/\text{AlBr}_3, \text{HNO}_3/\text{H}_2\text{SO}_4, \text{NaCN}, \text{H}_3\text{O}^+$

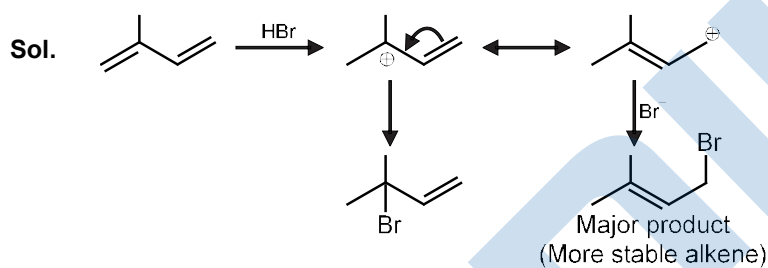
Ans. (3)



16. The major product formed in the following reaction is :

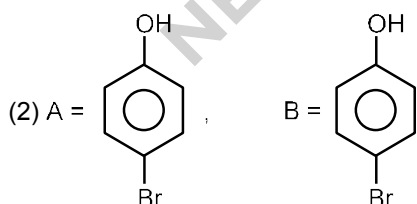
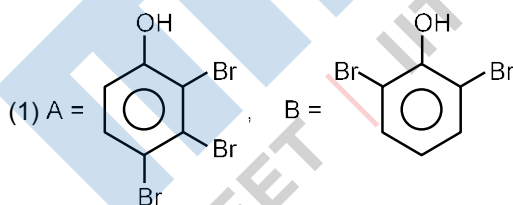
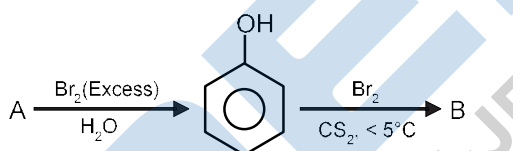


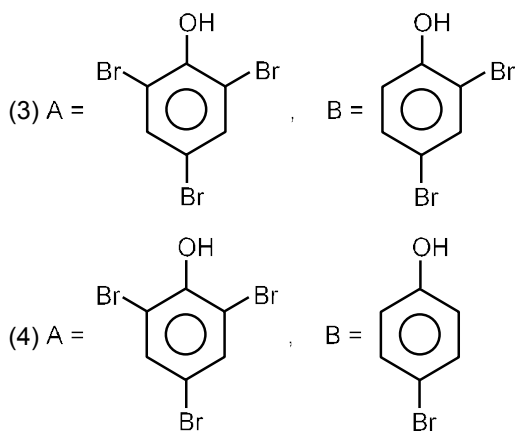
Ans. (2)



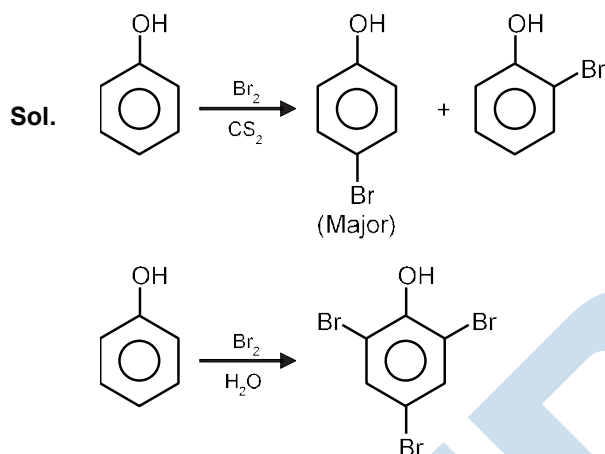
At 25°C thermodynamically controlled Product is formed as major product.

17. The correct options for the products A and B of the following reactions are :





Ans. (4)



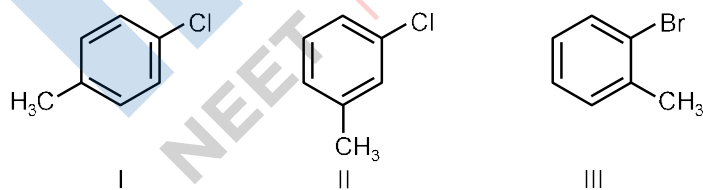
18. The polymer formed on heating Novolac with formaldehyde is :

- (1) Bakelite (2) Polyester (3) Melamine (4) Nylon 6, 6

Ans. (1)

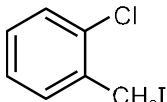
Sol. Note → Novolac on heating with formaldehyde undergoes cross linking to form infusible solid mass called Bakelite.

19. Among the following compounds I-IV, which one forms a yellow precipitate on reacting sequentially with (i) NaOH (ii) dil. HNO₃ (iii) AgNO₃ ?



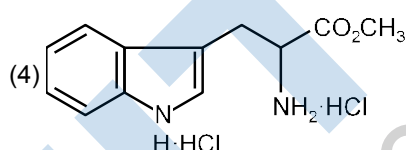
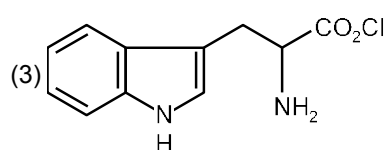
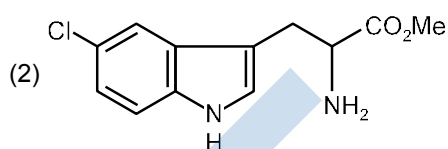
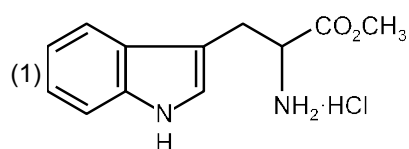
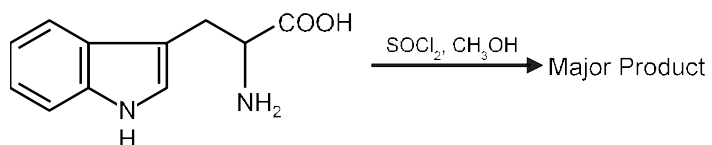
- (1) I (2) III (3) II (4) IV

Ans. (4)

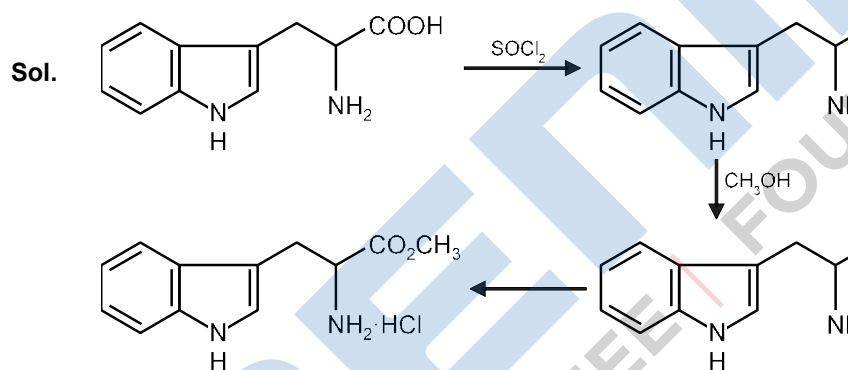
Sol. Only in , I^- can be substituted by OH^- , I^- combine with Ag^+ to form yellow ppt of AgI .

While in I, II and III substitution reaction is not possible due to partial double bond character.

20. The major product formed in the following reaction is :



Ans. (1)



Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The number of 4f electrons in the ground state electronic configuration of Gd^{2+} is.....

[Atomic number of Gd = 64]

Ans. (7)

Sol. $Gd = [Xe] 4f^7 5d^1 6s^2$

$Gd^{2+} = [Xe] 4f^8$ [due to orbital contraction]

2. An aqueous KCl solution of density 1.20 g mL^{-1} has a molality of 3.30 mol kg^{-1} . The molarity of the solution in mol L^{-1} is.....

Ans. (3)

Sol.
$$\text{Molality}(m) = \frac{M \cdot 1000}{1000 \cdot d \cdot M_{\text{solute}}}$$

$$3.3 = \frac{M \cdot 1000}{1000 \cdot 1.20 \cdot M}$$

$$3960 - 245.85 M = 1000 M$$

$$1245.85 M = 3960$$

$$M = \frac{3960}{1245.85} = 3.17 \approx 3M$$

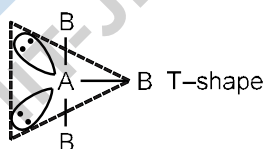
3. AB_3 is an interhalogen T-shaped molecule. The number of lone pairs of electrons on A is.....

Ans. (2)

Sol. In interhalogen compound AB_3

Central atom A is halogen so it contain 7 electrons in its outer most shell.

so its structure is



So central atom A contain 2 lone pairs of electrons.

4. Of the following four aqueous solutions, total number of those solutions whose freezing point is lower than that of $0.10 \text{ M C}_2\text{H}_5\text{OH}$

- (i) $0.10 \text{ M Ba}_3(\text{PO}_4)_2$ (ii) $0.10 \text{ M Na}_2\text{SO}_4$ (iii) 0.10 M KCl (iv) $0.10 \text{ M Li}_3\text{PO}_4$

Ans. (4)

Sol. $\Delta T_f = i k_f m$

As m is same for all solution so ΔT_f depends on vantHoff factor (i).

Greater is i small is freezing point.

Compound	i
C ₂ H ₅ OH	1
Ba ₃ (PO ₄) ₂	5
Na ₂ SO ₄	3
KCl	2
Li ₃ PO ₄	4

So the given solutions have lower freezing point than ethanol.

5. The Born-Haber cycle for KCl is evaluated with the following data :

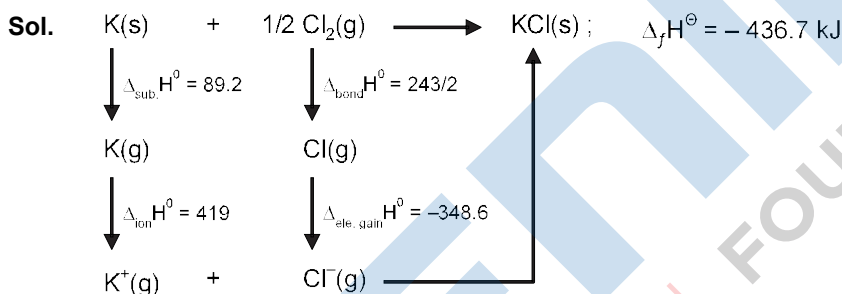
$$\Delta_f H^\ominus \text{ for KCl} = -436.7 \text{ kJ mol}^{-1}; \Delta_{\text{sub}} H^\ominus \text{ for K} = 89.2 \text{ kJ mol}^{-1};$$

$$\Delta_{\text{ionization}} H^\ominus \text{ for K} = 419 \text{ kJ mol}^{-1}; \Delta_{\text{electron gain}} H^\ominus \text{ for Cl(g)} = -348.6 \text{ kJ mol}^{-1};$$

$$\Delta_{\text{bond}} H^\ominus \text{ for Cl}_2 = 243.0 \text{ kJ mol}^{-1}$$

The magnitude of lattice enthalpy of KCl in kJ mol⁻¹ is.....

Ans. (718)



$$\text{so, } \Delta_f H^\ominus = \Delta_f H^\ominus (\text{KCl}) = \Delta_{\text{sub}} H^\ominus + \Delta_{\text{1st ionisation}} H^\ominus (\text{K}) + 1/2 \Delta_{\text{bond}} H^\ominus (\text{Cl}_2(\text{g})) + \Delta_{\text{electron gain}} H^\ominus + \Delta_{\text{lattice}} H^\ominus$$

$$-436.7 = 89.2 + 419 + 1/2(243) + (-348.6) + \Delta_{\text{lattice}} H^\ominus$$

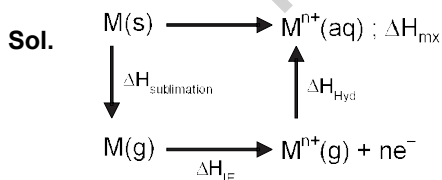
$$\Delta_{\text{lattice}} H^\ominus = -717.8 \text{ kJ mole}^{-1} \approx -718 \text{ kJ mole}^{-1}$$

6. These are physical properties of an element

- (A) Sublimation enthalpy
- (B) Ionisation enthalpy
- (C) Hydration enthalpy
- (D) Electron gain enthalpy

The total number of above properties that affect the reduction potential is.....

Ans. (3)



So reduction potential depends of sublimation, ionization and Hydration energy.

7. The ratio of number of water molecules in Mohr's salt and potash alum is..... $\times 10^{-1}$.

Ans. (5)

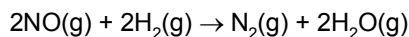
Sol. Mohr's salt = $\text{FeSO}_4(\text{NH}_4)_2\text{SO}_4 \cdot 6\text{H}_2\text{O}$

Potash alum = $\text{K}_2\text{SO}_4 \cdot \text{Al}_2(\text{SO}_4)_3 \cdot 24\text{H}_2\text{O}$

So water molecule ratio $\frac{6}{24} \frac{1}{4} = 0.25 = 2.5 \times 10^{-1}$

Note : NTA ans. is 5, but Reso ans. is 2.5.

8. The following data was obtained for chemical reaction given below at 975 K.



[NO]	[H ₂]	Rate
mol L ⁻¹	mol L ⁻¹	mol L ⁻¹ s ⁻¹
(A) 8×10^{-5}	8×10^{-5}	7×10^{-9}
(B) 24×10^{-5}	8×10^{-5}	2.1×10^{-8}
(C) 24×10^{-5}	32×10^{-5}	8.4×10^{-8}

The order of the reaction with respect to NO is.....

Ans. (1)

Sol. rate = $K[\text{NO}]^x [\text{H}_2]^y$

From (A) $7 \times 10^{-9} = K(8 \times 10^{-5})^x (8 \times 10^{-5})^y$ (1)

From (B) $2.1 \times 10^{-8} = K(24 \times 10^{-5})^x (8 \times 10^{-5})^y$ (2)

(1)/(2) $\Rightarrow \frac{7 \times 10^{-9}}{2.1 \times 10^{-8}} = \frac{K(8 \times 10^{-5})^x (8 \times 10^{-5})^y}{K(24 \times 10^{-5})^x (8 \times 10^{-5})^y}$

$$\frac{1}{3} = \frac{1}{3}^x$$

$\Rightarrow x = 1$

Order of reaction w.r.t. NO is $x = 1$.

9. The OH⁻ concentration in a mixture of 5.0 mL of 0.0504 M NH₄Cl and 2 mL of 0.0210 M NH₃ solution is $x \times 10^{-6}$ M. The value of x is.....

[Given $K_w = 1 \times 10^{-14}$ and $k_b = 1.8 \times 10^{-5}$]

Ans. (3)

Sol. $[\text{NH}_4\text{Cl}] = [\text{NH}_4^+] = 0.0504 \text{ M } V = 5.0 \text{ mL}$

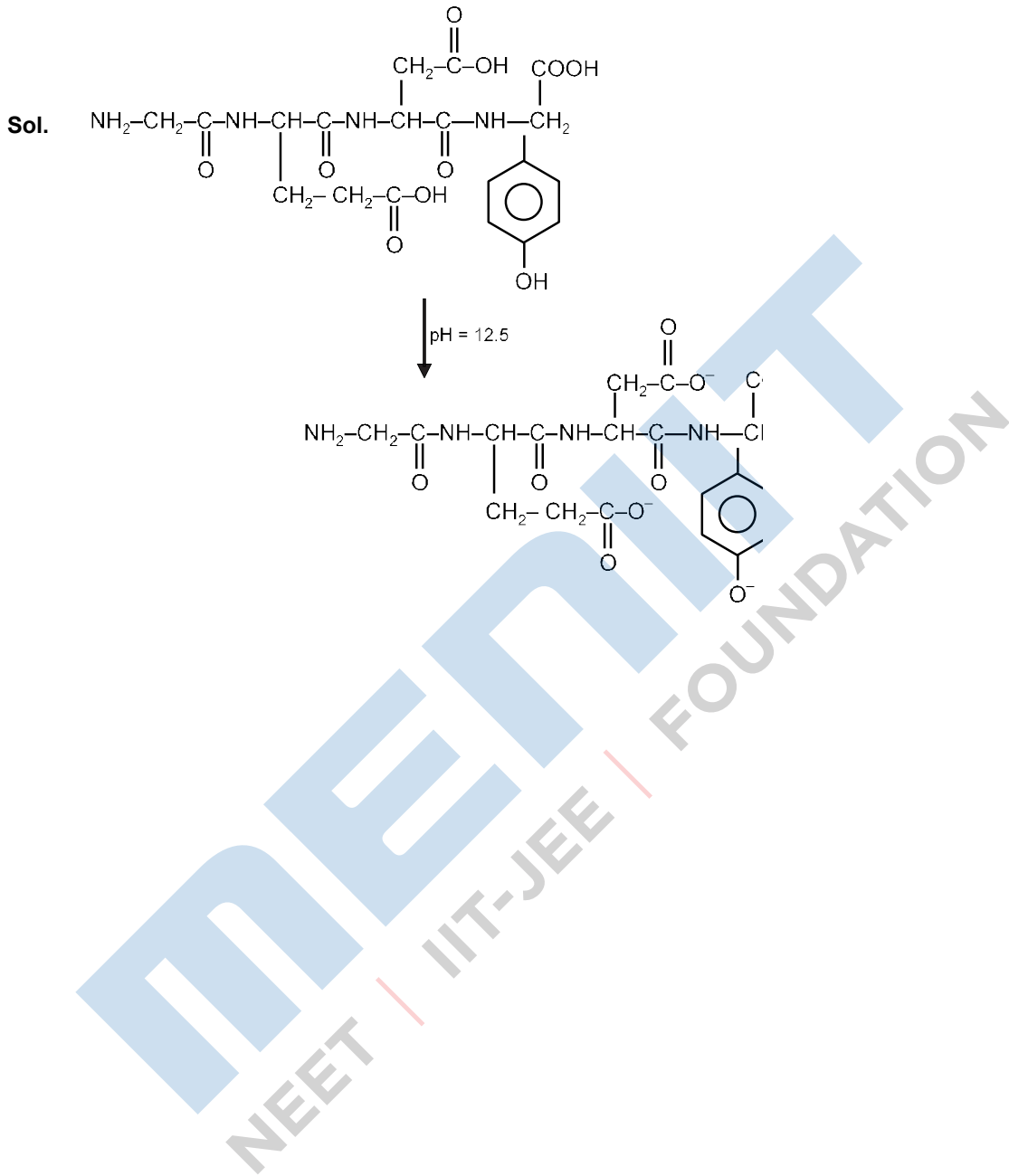
$[\text{NH}_3] = 0.0210 \text{ M } V = 2.0 \text{ mL}$

$$K_b = \frac{[\text{NH}_4][\text{OH}^-]}{[\text{NH}_3]}$$

$$[\text{OH}^-] = \frac{K_b [\text{NH}_3]}{[\text{NH}_4^+]} = \frac{1.8 \times 10^{-5} \times 0.0210}{0.0504} = 7.2 \times 10^{-6} \text{ M}$$

10. The total number of negative charge in the tetrapeptide, Gly-Glu-Asp-Tyr, at pH 12.5 will be.....

Ans. (4)



PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The sum of solutions of the equation $\frac{\cos x}{1 - \sin x} = |\tan 2x|$, $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$ is :
- (1) $\frac{10}{10}$ (2) $\frac{7}{30}$ (3) $\frac{1}{15}$ (4) $\frac{11}{30}$

Ans. (4)

Sol. $\frac{\cos x}{1 - \sin x} = |\tan 2x|$ $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$

Case-I $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ & $x \in \left(\frac{3\pi}{4}, \pi\right)$

$$\frac{\cos x}{1 - \sin x} = \frac{2 \sin x \cos x}{\cos^2 x - \sin^2 x} \Rightarrow \cos x (\cos^2 x - 2 \sin^2 x - 1) = 0$$

$\cos x = 0$ & $4 \sin^2 x + 2 \sin x - 1 = 0$

$\sin x = \frac{2 - 2\sqrt{5}}{8}, \frac{1 - 2\sqrt{5}}{8}, \frac{1 + \sqrt{5}}{4}$

$x = \frac{3\pi}{10}, \frac{9\pi}{10}$

Case-II $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ & $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ & $x = 0$

$\cos x = 0$ & $\sin x = \frac{1}{2}$ & $x = \frac{\pi}{6}$

sum of solutions $\frac{3\pi}{10} + \frac{9\pi}{10} + \frac{\pi}{6} = \frac{11\pi}{30}$

2. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set :
- (1) {80, 83, 86, 89} (2) {84, 87, 90, 93} (3) {79, 81, 83, 85} (4) {84, 86, 88, 90}

Ans. (3)

Sol. Let H and L is set of people suffered from heart ailment and lungs infections respectively.

$n(H) = 89\%$

$n(L) = 98\%$

$n(H \cap L) = x\%$

$\max\{0, n(H) + n(L) - n(H \cup L)\} \leq n(H \cap L) \leq \min\{n(H), n(L)\}$

So, $87\% \leq x \leq 89\%$

Ans. (1)

Sol. Put $x = \sin^2\theta, 0 < x < 1$

$$\sin \theta = \sqrt{x}$$

$$f(x) = \cos 2 \tan^{-1} \sin \theta = \cot^{-1} \sqrt{\frac{1 - \sin^2 \theta}{\sin^2 \theta}}$$

$$\Rightarrow f(x) = \cos(2 \tan^{-1}(\sin \theta))$$

$$f(x) = \cos(2 \tan^{-1} \sqrt{x})$$

$$\frac{1 - \tan^2(\tan^{-1} \sqrt{x})}{1 + \tan^2(\tan^{-1} \sqrt{x})}$$

$$f(x) = \frac{1 - x}{1 + x}$$

$$f'(x) = \frac{(1-x)(-1) - (1+x)(1)}{(1+x)^2} = \frac{-2}{(1+x)^2}$$

$$(1-x)^2 f'(x) + 2f(x)^2 = 0 \text{ option (1) satisfied}$$

6. The sum of the series $\frac{1}{x-1} - \frac{2}{x^2-1} + \frac{2^2}{x^4-1} - \dots + \frac{2^{100}}{x^{2^{100}}-1}$ when $x = 2$ is :

- (1) $1 - \frac{2^{101}}{4^{101}-1}$ (2) $1 - \frac{2^{100}}{4^{100}-1}$ (3) $1 - \frac{2^{101}}{4^{101}-1}$ (4) $1 - \frac{2^{100}}{4^{100}-1}$

Ans. (Bonus)

Sol. Adding and subtracting $\frac{1}{x-1}$

$$S = \frac{1}{x-1} - \frac{1}{x-1} + \frac{1}{x-1} - \frac{2}{x^2-1} + \frac{2^2}{x^4-1} - \dots + \frac{2^{100}}{x^{2^{100}}-1}$$

$$S = \frac{1}{x-1} - \frac{2}{x^2-1} + \frac{2}{x^2-1} - \frac{2^2}{x^4-1} + \dots + \frac{2^{100}}{x^{2^{100}}-1}$$

$$S = \frac{1}{x-1} - \frac{2^{101}}{x^{2^{101}}-1}$$

$$\text{For } x = 2 \Rightarrow S = 1 - \frac{2^{101}}{2^{2^{101}}-1}$$

7. The value of $\lim_{n \rightarrow \infty} \frac{1}{n} \frac{2n-1}{n^2-4n^2}$ is :

- (1) $\frac{1}{2} \tan^{-1}(4)$ (2) $\frac{1}{4} \tan^{-1}(4)$ (3) $\tan^{-1}(4)$ (4) $\frac{1}{2} \tan^{-1}(2)$

Ans. (1)

$$0 \quad 30m \quad \frac{30}{4m}$$

$$4m^2 = 1$$

$$m = \frac{1}{2}$$

$$\text{for } m = \frac{1}{2}$$

$$y = \frac{x}{2} - 15$$

$$x - 2y + 30 = 0$$

$$\text{Length of OB} = \frac{15 \cdot 0 + 30}{\sqrt{5}} = 3\sqrt{5}, \text{ radius of circle} = \frac{15}{2}$$

$$\text{Length of BC} = \sqrt{\frac{225}{4} + 45} = 3\sqrt{5} \quad \text{BC} = \frac{3\sqrt{5}}{2}$$

$$\text{Length of chord AC} = 2 \cdot \frac{3\sqrt{5}}{2} = 3\sqrt{5}$$

$$\text{similarly for } m = \frac{1}{2} \text{ length of chord AC} = 3\sqrt{5}$$

10. A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$, and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following point lies on P ?

- (1) (1, 0, 1) (2) (0, 1, 1) (3) (-1, 1, 2) (4) (2, -1, 1)

Ans. (2)

Sol. A plane which containing the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$ is

$$(x + 2y + 3z + 1) + k(x - y - z - 6) = 0$$

$$\Rightarrow (1 + k)x + (2 - k)y + (3 - k)z + 1 - 6k = 0$$

This plane is perpendicular to the plane $-2x + y + z + 8 = 0$ then

$$-2(1 + k) + 1(2 - k) + 1(3 - k) = 0 \Rightarrow k = \frac{3}{4}$$

$$\text{Equation of plane is } \frac{7}{4}x + \frac{5}{4}y + \frac{9}{4}z + \frac{14}{4} = 0 \Rightarrow 7x + 5y + 9z = -14$$

(0, 1, 1) lies on the plane

11. The value of $\int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \frac{x-1}{x-1} \cdot \frac{x-1}{x-1} \cdot 2^{\frac{1}{2}} dx$ is :

- (1) $\log_e 4$ (2) $\log_e 16$ (3) $2 \log_e 16$ (4) $4 \log_e 3 + 2\sqrt{2}$

Ans. (2)

Sol.

$$\frac{1}{\sqrt{2}} \int \sqrt{\frac{x-1}{x+1} \cdot \frac{x-1}{x+1}} dx$$

$$\frac{1}{\sqrt{2}} \int \sqrt{\frac{4x^2}{x^2-1}} dx$$

$$\frac{1}{\sqrt{2}} \int \sqrt{\frac{16x^2}{1-x^2}} dx$$

$$\frac{1}{\sqrt{2}} \int \frac{4|x|}{1-x^2} dx$$

$$8 \int_0^1 \frac{x}{1-x^2} dx = 4 \ln |1-x^2|_0^1 = \log_e 16$$

12. Let A and B be independent events such that $P(A) = p$, $P(B) = 2p$, The largest value of p, for which P (exactly one of A, B occurs) $\frac{5}{9}$ is :

- (1) $\frac{5}{12}$ (2) $\frac{1}{3}$ (3) $\frac{4}{9}$ (4) $\frac{2}{9}$

Ans. (1)

Sol. Probability (exactly one of A and B) $\frac{5}{9}$

$$P(A) - P(B) - 2P(A)P(B) = \frac{5}{9}$$

$$p - 2p - 4p^2 = \frac{5}{9}$$

$$36p^2 - 27p + 5 = 0$$

$$\Rightarrow (12p - 5)(3p - 1) = 0$$

$$p = \frac{1}{3} \text{ or } \frac{5}{12}$$

13. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum_{r=0}^{20} r^2 \cdot {}^{20}C_r$ is equal to

- (1) 420×2^{18} (2) 380×2^{19} (3) 380×2^{18} (4) 420×2^{19}

Ans. (1)

Sol.
$$\sum_{r=0}^{20} r^2 \binom{20}{r} = \sum_{r=1}^{20} r \binom{20}{r} = \sum_{r=1}^{20} r \cdot 20 \binom{19}{r-1}$$

$$= 20 \sum_{r=0}^{19} (r+1) \binom{19}{r} = 20 \sum_{r=0}^{19} r \binom{19}{r} + 20 \sum_{r=0}^{19} \binom{19}{r}$$

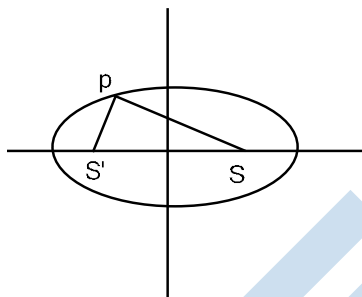
$$= 20 \cdot 19 \sum_{r=1}^{19} \binom{18}{r-1} + 20 \cdot 2^{19}$$

$$\Rightarrow (380)2^{18} + 20 \cdot 2^{19} \Rightarrow 2^{20} \cdot (95 + 10) = (105)2^{20} = 420 \times 2^{18}$$

- 14.** On the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x + 2y = 0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5 - e^2) \cdot A$ is
- (1) 12 (2) 6 (3) 14 (4) 24

Ans. (2)

Sol. $p \sqrt{8} \cos \theta, 2 \sin \theta$



Equation of tangent at point p

$$\frac{x}{\sqrt{8}} \cos \theta + \frac{y}{2} \sin \theta = 1 \quad 0$$

Slope $\frac{1}{\sqrt{2}} \cot \theta = -2$

$$\cot \theta = -2\sqrt{2} \quad \cos \theta = \frac{2\sqrt{2}}{3}, \sin \theta = \frac{1}{3}$$

So, point P $(-\frac{8}{3}, \frac{2}{3})$

$$A = \frac{1}{2} \cdot 2ae \cdot \frac{2}{3} = \frac{4}{3} e = \frac{1}{\sqrt{2}}$$

So, $(5 - e^2) A = 5 \cdot \frac{1}{2} = \frac{5}{2} = 2.5$

15. Let $\theta \in (0, \frac{\pi}{2})$. If the system of linear equations.

$$(1 + \cos^2\theta)x + \sin^2\theta y + 4 \sin 3\theta z = 0$$

$$\cos^2\theta x + (1 + \sin^2\theta)y + 4 \sin 3\theta z = 0$$

$$\cos^2\theta x + \sin^2\theta y + (1 + 4 \sin 3\theta)z = 0$$

has a non-trivial solution, then the value of θ is :

- (1) $\frac{1}{18}$ (2) $\frac{5}{18}$ (3) $\frac{4}{9}$ (4) $\frac{7}{18}$

Ans. (4)

Sol.
$$\begin{vmatrix} 1 & \cos^2 & \sin^2 & 4 \sin 3 \\ \cos^2 & 1 & \sin^2 & 4 \sin 3 \\ \cos^2 & \sin^2 & 1 & 4 \sin 3 \end{vmatrix} = 0$$

$$R_3 \rightarrow R_3 - R_2$$

$$\begin{vmatrix} 1 & \cos^2 & \sin^2 & 4 \sin 3 \\ \cos^2 & 1 & \sin^2 & 4 \sin 3 \\ 0 & 1 & 1 & 1 \end{vmatrix}$$

$$C_2 \rightarrow C_2 + C_3$$

$$\begin{vmatrix} 1 & \cos^2 & \sin^2 & 4 \sin 3 & 4 \sin 3 \\ \cos^2 & 1 & \sin^2 & 4 \sin 3 & 4 \sin 3 \\ 0 & 1 & 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 2(1+2\sin 3\theta) = 0 \Rightarrow 2\sin 3\theta = -1$$

$$\sin 3\theta = -\frac{1}{2} \Rightarrow 3\theta = \frac{7\pi}{6}$$

16. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and β are the mean and standard respectively for correct data, then (α, β) is

- (1) (10.5, 26) (2) (10.5, 25) (3) (11, 25) (4) (11, 26)

Ans. (1)

Sol. $x_1 + x_2 + x_3 + \dots + x_{19} + 25 = 200$

$$\Rightarrow x_1 + x_2 + x_3 + \dots + x_{19} = 175$$

$$\text{New mean} = \frac{x_1 + x_2 + x_3 + \dots + x_{19} + 35}{20} = \frac{175 + 35}{20} = 10.5$$

$$\text{S.D.} = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2 + \dots + x_{19}^2 + (25)^2}{20} - (10)^2}$$

$$2.5 = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2 + \dots + x_{19}^2 + (25)^2}{20} - (10)^2}$$

$$x_1^2 \ x_2^2 \ x_3^2 \ \dots \ x_{19}^2 \ 1500$$

$$\text{New S.D.} = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2 + \dots + x_{19}^2 + (35)^2}{20}} \quad (10.5)^2$$

$$\sqrt{\frac{1500 + (35)^2}{20}} \quad (110.25)$$

$$\sqrt{26}$$

$$(\alpha, \beta) = (10.5, 26)$$

17. If the sum of an infinite GP a, ar, ar^2, ar^3, \dots is 15 and the sum of the squares of each term is 150, then the sum of ar^2, ar^4, ar^6, \dots is :

(1) $\frac{25}{2}$

(2) $\frac{9}{2}$

(3) $\frac{5}{2}$

(4) $\frac{1}{2}$

Ans. (4)

Sol. Given $\frac{a}{1-r} = 15$ (1)

and $\frac{a^2}{1-r^2} = 150 \implies \frac{a}{1-r} \cdot \frac{a}{1+r} = 150 \implies \frac{a}{1+r} = 10$ (2)

solving equation (1) and (2)

$$r = \frac{1}{5} \therefore a = 12 \text{ from equation (1)}$$

Now, $ar^2 + ar^4 + ar^6 + \dots \infty$

$$\frac{ar^2}{1-r^2} = \frac{12 \cdot \frac{1}{25}}{1 - \frac{1}{25}} = \frac{1}{2}$$

18. If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$, and $Q = A^T B A$, then the inverse of the matrix

$A Q^{2021} A^T$ is equal to :

(1) $\begin{pmatrix} 1 & 2021i \\ 0 & 1 \end{pmatrix}$

(2) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

(3) $\begin{pmatrix} \frac{1}{\sqrt{5}} & 2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$

(4) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

Ans. (4)

Sol. $AA^T = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{5}} & 2 \\ \frac{2}{\sqrt{5}} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$$Q = A^T B A$$

$$\Rightarrow Q^2 = (A^T B A) (A^T B A) = A^T B^2 A$$

$$\Rightarrow Q^3 = A^T B^3 A$$

$$\Rightarrow Q^{2021} = A^T B^{2021} A$$

Now let $P = A Q^{2021} A^T$

$$P = A (A^T B^{2021} A) A^T$$

given $A A^T = I$

$$P = B^{2021}$$

$$B^2 = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ i & 1 & i & 1 & 2i & 1 \end{pmatrix}$$

$$B^3 = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 2i & 1 & i & 1 & 3i & 1 \end{pmatrix}$$

.....

$$B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$$

inverse of $p \quad p^{-1} \quad B^{2021}^{-1} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

19. If the truth value of the Boolean expression $((p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)) \rightarrow (p \wedge q)$ is false, then the truth value of the statements p, q, r respectively can be :

- (1) T F T (2) T F F (3) F F T (4) F T F

Ans. (2)

Sol.

p	q	r	$p \vee q$	$q \rightarrow r$	$\sim r$	$(p \vee q) \wedge (q \rightarrow r) \wedge \sim r$	$p \wedge q$	$((p \vee q) \wedge (q \rightarrow r) \wedge \sim r) \rightarrow p \wedge q$
T	T	T	T	T	F	F	T	T
T	T	F	T	F	T	F	T	T
T	F	T	T	T	F	F	F	T
T	F	F	T	T	T	T	F	F
F	T	T	T	T	F	F	F	T
F	T	F	T	F	T	F	F	T
F	F	T	F	T	F	F	F	T
F	F	F	F	T	T	F	F	T

20. Let $y = y(x)$ be a solution curve of the differential equation $(y + 1) \tan^2 x \, dx + \tan x \, dy + y \, dx = 0$,
 $x \in (0, \frac{\pi}{2})$, if $\lim_{x \rightarrow 0} xy(x) = 1$, then the value of $y \left(\frac{\pi}{4} \right)$ is :

- (1) $\frac{1}{4}$ (2) $\frac{1}{4} - 1$ (3) $\frac{1}{4}$ (4) $\frac{1}{4}$

Ans. (3)

Sol. $\frac{dy}{dx} + (1 + y) \tan x = -y \cot x$

$$\frac{dy}{dx} + y(\tan x + \cot x) = -\tan x$$

I.F. $e^{\int (\tan x + \cot x) dx}$

$$e^{\int \frac{\tan^2 x + 1}{\tan x} dx}$$

$$= \tan x$$

$$y \tan x + \tan^2 x \, dx = c$$

$$y \tan x + (1 + \sec^2 x) dx = c$$

$$y \cdot \tan x = x - \tan x + c$$

$$\lim_{x \rightarrow 0} xy = 1 \implies \lim_{x \rightarrow 0} \frac{x}{\tan x} (x - \tan x + c) = 1 \implies 1(0 - 0 + c) = 1 \implies c = 1$$

Then the function is $y \cdot \tan x = x - \tan x + 1$

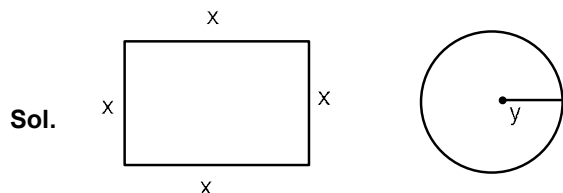
$$y \left(\frac{\pi}{4} \right) = \frac{\pi}{4} - 1 + 1 \implies y \left(\frac{\pi}{4} \right) = \frac{\pi}{4}$$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. A wire of length 36m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle, If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\frac{4}{\pi} k$ is equal to _____.

Ans. (36)



$$4x + 2\pi y = 36 \text{ (given)}$$

$$\text{Area} = x^2 + \pi y^2$$

$$A = x^2 - \frac{1}{\pi}(18 - 2x)^2$$

$$\frac{dA}{dx} = 2x - \frac{2}{\pi}(18 - 2x)(-2)$$

$$\frac{dA}{dx} = \frac{2}{\pi} x - (36 - 4x) = 0$$

$$\frac{dA}{dx} = 0 \implies 2x - 36 + 4x = 0 \implies 6x = 36 \implies x = 6$$

$$K \text{ [circumference of circle]} = 2\pi y \implies K = \frac{36}{\pi} - 4x$$

$$\text{Value of } \frac{4}{\pi} k = 36$$

2. Let $a, b \in \mathbb{R}, b \neq 0$. Define a function

$$f(x) = \begin{cases} a \sin \frac{x}{2} - 1, & \text{for } x < 0 \\ \frac{\tan 2x - \sin 2x}{bx^3}, & \text{for } x > 0 \end{cases}$$

If f is continuous at $x = 0$, then $10 - ab$ is equal to

Ans. (14)

Sol. If function is continuous at $x = 0$

$$\text{then, L.H.L} = \text{R.H.L} = f(0)$$

$$\text{L.H.L} = -a$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} \frac{\tan 2x - \sin 2x}{bx^3}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$$

$$\lim_{x \rightarrow 0} \frac{\frac{(2x)^3}{3} - \frac{(2x)^3}{6}}{bx^3} = \frac{\frac{8}{3} - \frac{8}{6}}{b} = \frac{4}{b}$$

$$f(0) = a \sin \frac{\pi}{2} = a \Rightarrow a = \frac{4}{b}$$

$$\Rightarrow \text{L.H.L.} = \text{R.H.L.} = f(0) \Rightarrow a = \frac{4}{b} \Rightarrow -ab = 4 \Rightarrow 10 - ab = 14$$

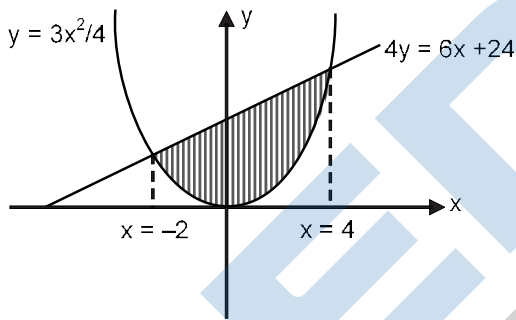
3. The area of the region $S = \{(x, y) : 3x^2 \leq 4y \leq 6x + 24\}$ is

Ans. (27)

Sol. $S = \{(x, y) : 3x^2 \leq 4y \leq 6x + 24\}$

$$y = \frac{3x^2}{4} \quad 4y = 6x + 24$$

area of the region



$$A = \int_{-2}^4 \left(\frac{6x + 24}{4} - \frac{3x^2}{4} \right) dx$$

$$A = \left[\frac{3}{4}x^2 + 6x - \frac{3x^3}{4} \right]_{-2}^4$$

$$A = 20 - (-7) = 27 \text{ square unit}$$

4. The sum of all integral values of k ($k \neq 0$) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is

Ans. (66)

$$\text{Sol. } \frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k} \Rightarrow \frac{2x-4-x-1}{(x-1)(x-2)} = \frac{2}{k} \Rightarrow 2x^2 - (6+k)x + 3k + 4 = 0$$

for no real roots $D < 0$

$$\Rightarrow (6+k)^2 - 8(3k+4) < 0 \Rightarrow k^2 + 12k + 36 - 24k - 32 < 0 \Rightarrow (k-6)^2 - 32 < 0$$

Integral value of k = 1,2,3,4,5,6,7,8,9,10,11

Sum = 66

5. Let the line L be the projection of the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$ in the plane $x - 2y - z = 3$. If d is the distance of the point (0,0,6) from L, then d^2 is equal to

Ans. (26)

Sol. Any point on the line $P(2r + 1, r + 3, 2r + 4)$
foot of perpendicular on the plane

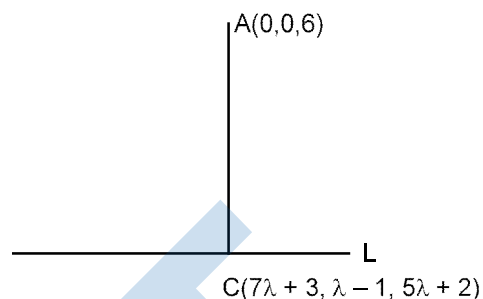
$$\frac{x - (2r + 1)}{1} = \frac{y - (r + 3)}{2} = \frac{z - (2r + 4)}{1} = \frac{r - 6}{3}$$

$$x = \frac{7r + 9}{3}; y = \frac{r + 3}{3}; z = \frac{5r + 6}{3}$$

$$\Rightarrow \text{Projection line L : } \frac{x - 3}{7} = \frac{y - 1}{1} = \frac{z - 2}{5} \quad \dots\dots(i)$$

Direction ratio of AC are $7\lambda + 3, \lambda - 1, 5\lambda - 4$

Now AC is perpendicular to the projection line (i) So, $\lambda = 0 \Rightarrow C(3, -1, 2) \Rightarrow d^2 = AC^2 = 26$



6. The number of three –digit even numbers, formed by the digits 0,1,3,4,6,7 if the repetition of digits is not allowed, is

Ans. (52)

Sol. 0,1,3,4,6,7

The number of three –digit even numbers ending with 0 = $5 \times 4 = 20$

The number of three –digit even numbers ending with 4 = $4 \times 4 = 16$

The number of three –digit even numbers ending with 6 = $4 \times 4 = 16$

Total three digits even numbers = $20 + 16 + 16 = 52$

7. If $y = y(x)$ is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is equal to

Ans. 40

Sol. when $x = 0$ then $y = 1$

$$\ln(x + y) = 4xy$$

$$\Rightarrow x + y = e^{4xy}$$

Now differentiate

$$\Rightarrow 1 + y' = e^{4xy} (4y + 4xy'') \quad \dots\dots (i)$$

$$\text{at } (0, 1) \Rightarrow y'(0) + 1 = 4 \Rightarrow y'(0) = 3$$

No, again differentiate equation (i)

$$\Rightarrow y''(0) = e^{4xy} (4y + 4xy'')^2 + e^{4xy} (4y' + 4y' + 4xy''')$$

$$\text{at } (0, 1)$$

$$\Rightarrow y'(0) = 1(4 \times 1 + 0)^2 + 1(4 \times 3 + 4 \times 3 + 0)$$

$$\Rightarrow y'(0) = 16 + 24 = 40$$

$$\Rightarrow y''(0) = 40$$

8. Let $z = \frac{1 - i\sqrt{3}}{2}, i, \sqrt{-1}$. Then the value of $z + z^2 + z^3 + \dots + z^{21}$ is

Ans. (13)

Sol. $z = -\omega$

$$\text{Let } k = z + z^2 + z^3 + \dots + z^{21}$$

$$k = \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \dots + \frac{1}{z^{21}}$$

$$k = -8$$

$$\Rightarrow 21 + k = 21 - 8$$

$$= 13$$

9. If ${}^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + \dots + 15 \cdot {}^{15}P_{15} = {}^qP_r - s, 0 \leq s \leq 1$, then ${}^{q+s}C_{r-s}$ is equal to.....

Ans. (136)

Sol. ${}^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + \dots + 15 \cdot {}^{15}P_{15}$

$$= 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + 15 \cdot 15! = 1! + (3! \cdot 2!) + (4! \cdot 3!) + \dots + (16 \cdot 15!)$$

$$= 16! - 1! = {}^qP_r - s \Rightarrow q = r = 16 \text{ and } s = 1$$

$$\text{So, } {}^{q+s}C_{r-s} = {}^{17}C_{15} = \frac{17 \cdot 16}{2} = 136$$

10. The locus of a point, which moves such that the sum of squares of its distance from the points (0,0), (1,0), (0,1), (1,1) is 18 units, is a circle of diameter d. Then d^2 is equal to

Ans. 16

Sol. Let P (x,y)

$$x^2 + y^2 + (x-0)^2 + (y-1)^2 + (x-1)^2 + (y-0)^2 + (x-1)^2 + (y-1)^2 = 18$$

$$4x^2 + 4y^2 - 4x - 4y + 4 = 18$$

$$x^2 + y^2 - x - y - \frac{14}{4} = 0$$

$$\text{Centre } \left(\frac{1}{2}, \frac{1}{2}\right) \quad r = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{14}{4}} = 2$$

$$d = 2r = 2 \times 2 = 4$$

$$\text{So, } d^2 = 16$$